Module 2:

EULERIAN AND HAMILTONIAN GRAPHS

- Euler graphs
- Operations on Graphs
- Hamiltonian Paths & Circuits
- Travelling salesman problem
- Directed Graphs
- Types of digraphs
- Digraphs & Binary relation
- Directed paths
- Fleurys Algorithm.

Euler line

A closed walk in a graph that contain all the edges in the graph exactly once is called a Euler line.

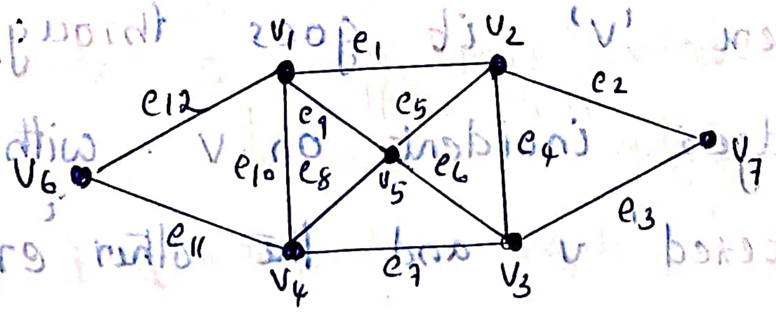
Euler Graph

A graph that consists of an Euler line is called an Euler graph.

Note:

Since the Euler line contains all the edges of the graph Euler graph is connected and do not have isolated vertex

Eg:



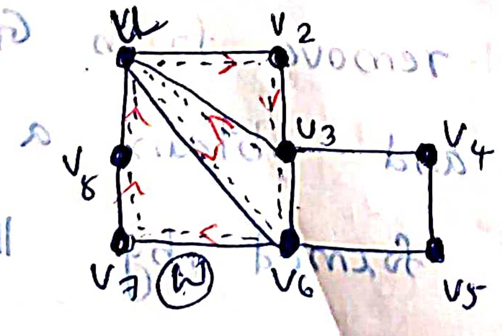
$v_1 e_1 v_2 e_2 v_7 e_3 v_3 e_4 v_2 e_5 v_5 e_6 v_3 e_7 v_4 e_8 v_5 e_9 v_1 e_{10} v_4 e_{11} v_6 e_{12} v_1$ is an Euler line in the graph.

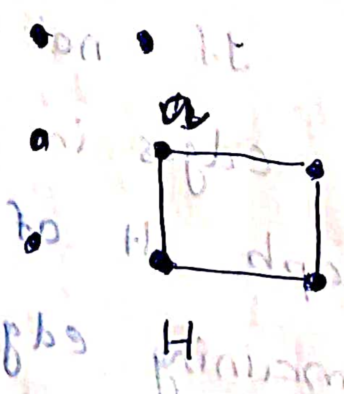
Hence it is an Euler graph

vertices of H are also even. Moreover H must touch W at least at one vertex 'a' because G is connected. Starting from 'a' we can construct a new walk in H terminating at 'a' and this walk in H can be combined with W to form a new walk which starts and ends at vertex v. and has more edges than W.

This process can be repeated untill we obtain a closed walk that travels through all edges of G exactly once.

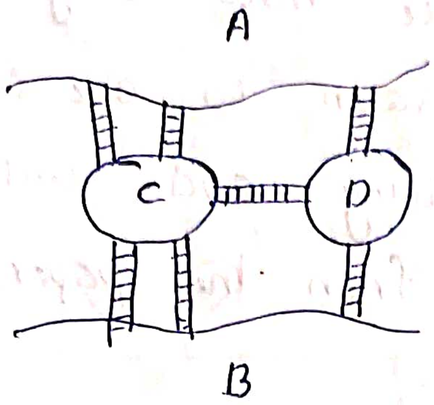
Thus G is an Euler graph.



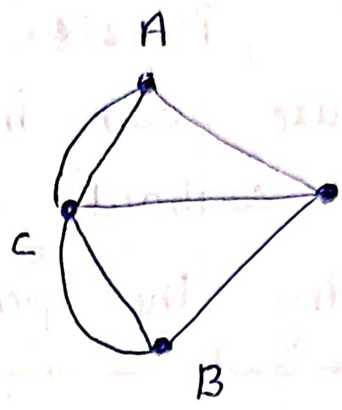


Application of Euler Graph

I Konigsberg Bridge Problem:



Graph



The problem was to start at any of the 4 land areas A, B, C, D walk over each of the seven bridges exactly once and return to the starting point.

Answer :

No such walk exists.

Reason :

Not all the vertices are of even degree. Hence the corresponding graph of Konigsberg bridge problem is not an Euler graph. ie, there does not exists a closed walk travelling through each edge exactly once.

II The concept of Euler graph is used in various puzzles.

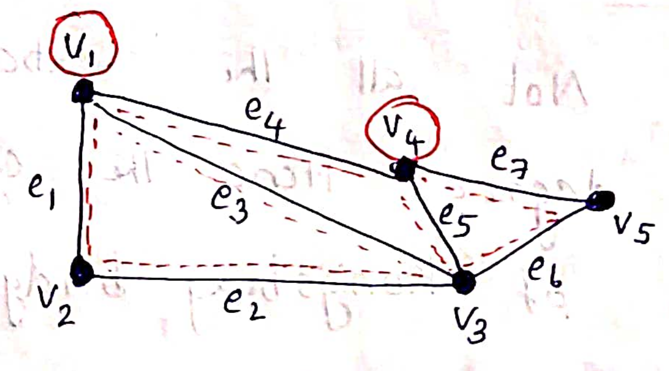
Puzzles like how a given picture can be drawn in one continuous line without retracing and without lifting the pencil from the paper.

Unicursal Graph

A graph G is called unicursal graph if it has an open walk consists of every edge of G exactly once (unicursal line).

i.e., a graph with an open Euler line.

Eg:



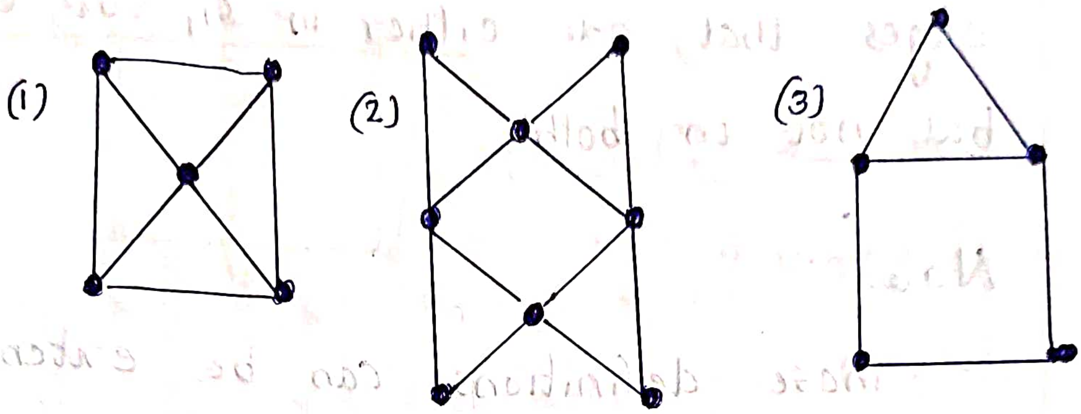
$v_1 e_1 v_2 e_2 v_3 e_3 v_1 e_4 v_4 e_5 v_3 e_6 v_5 e_7 v_4$

A unicursal line starting from vertex v_1 and ending on v_4

Note :

A connected graph is unicursal iff it has exactly two vertices of odd degree.

Qn.



Identify Euler & Unicursal graphs.

Operations on Graphs

I Union ($\cup$)

Union of two graphs $G_1 = (V_1, E_1)$ & $G_2 = (V_2, E_2)$ is another graph $G_3 = (V_3, E_3)$ where, $V_3 = V_1 \cup V_2$ & $E_3 = E_1 \cup E_2$

ie $G_3 = G_1 \cup G_2$

II Intersection ($\cap$)

$G_3 = G_1 \cap G_2$, where $V_3 = V_1 \cap V_2$ & $E_3 = E_1 \cap E_2$. ie, G_3 consists of only those vertices and edges that are both in G_1 & G_2.

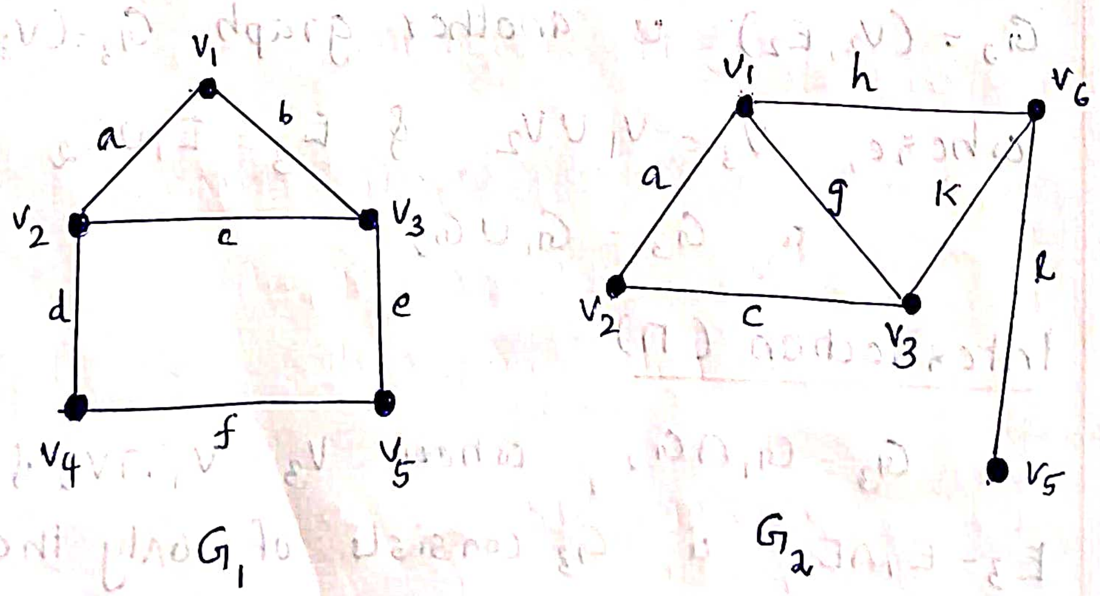
III **Ring sum ⊕**

The ring sum of G_1 and G_2 is denoted by $G_1 \oplus G_2$. It is a graph consisting of vertex set $V_1 \cup V_2$ and edges that are either in G_1 or G_2 but not in both.

Note:

- These definitions can be extended to any finite number of graphs.
- The operations $\cup$, $\cap$, & $\oplus$ are commutative.

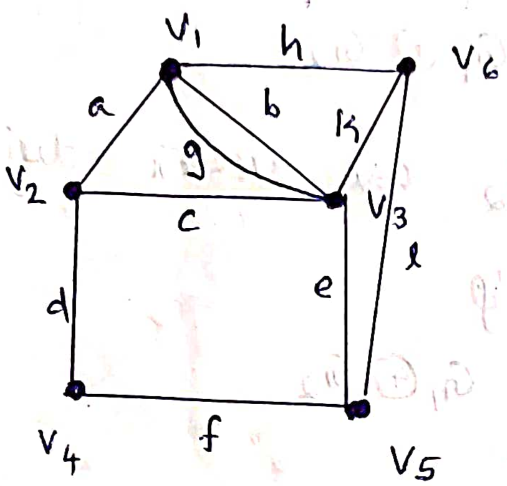
Qn. Find the union, intersection, and ring sum of the following graphs G_1 & G_2



$G_1 \cup G_2$

$V = V_1 \cup V_2 = \{v_1, v_2, v_3, v_4, v_5, v_6\}$

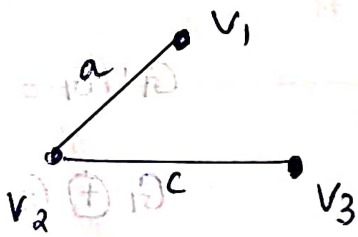
$E = E_1 \cup E_2 = \{a, b, c, d, e, f, g, h, k, l\}$



$G_1 \cap G_2$

$V = V_1 \cap V_2 = \{v_1, v_2, v_3, v_5\}$

$E = E_1 \cap E_2 = \{a, c\}$

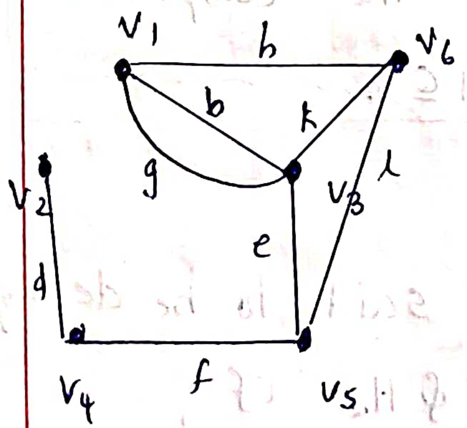


$G_1 \oplus G_2$

$V = V_1 \cup V_2 = \{v_1, v_2, v_3, v_4, v_5, v_6\}$

E = { edge either in G_1 or in G_2, not in both }

$= \{b, d, e, f, g, h, k, l\}$



$G_1 \oplus G_2$

Note:

1) If G_1 and G_2 are edge disjoint

then, $G_1 \cap G_2$ = Null graph

$G_1 \cup G_2 = G_1 \oplus G_2$

2) If G_1 and G_2 are vertex disjoint

then, $G_1 \cap G_2 = \phi$

$G_1 \cup G_2 = G_1 \oplus G_2$

3) For any graph G,

$G \cup G = G \cap G = G$

$G \oplus G$ = Null graph

4) If H is any subgraph of G, then

$G \oplus H = G - H$

$\therefore$ $G \oplus H$ is called the compliment of H in G whenever $H \subseteq G$.

IV **DECOMPOSITION**

A graph G is said to be decomposed into two subgraphs H_1 & H_2 if,

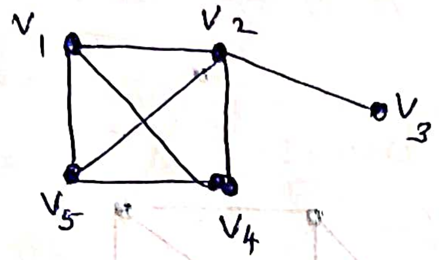
$H_1 \cup H_2 = G$

$H_1 \cap H_2$ = a null graph.

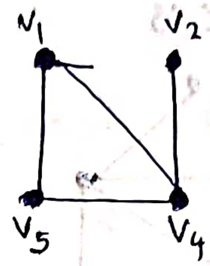
In other words, every edge of G occurs either in H_1 or in H_2 but not in both.

(some vertices may occur both in H_1 & H_2)

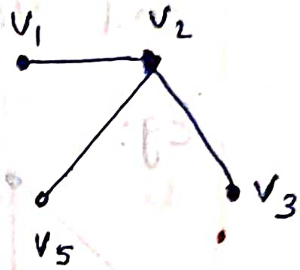
Eg:



G



H_1



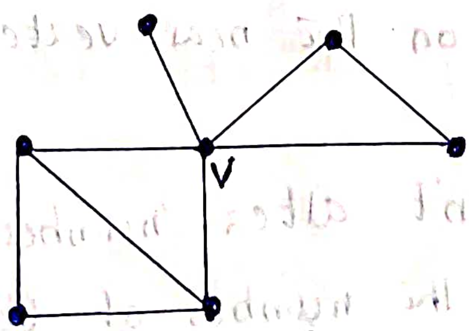
H_2

V DELETION

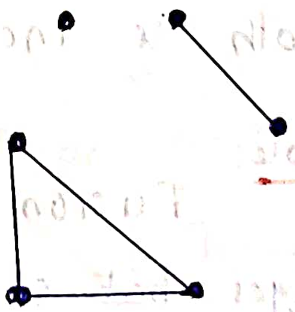
Vertex deletion:

If V is a vertex in graph G then G-V denote a subgraph of G obtained by deleting V from G. Deletion of a vertex always implies the deletion of all edges incident on that vertex

Eg:



G.

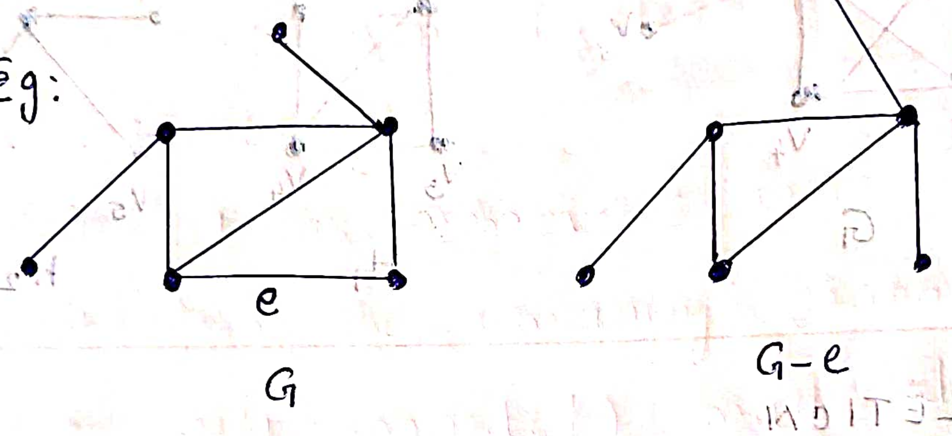


G-V

Edge Deletion

If 'e' is an edge in G, then G−e is a subgraph of G obtained by deleting 'e' from G. Deletion of an edge doesn't imply deletion of its end vertices.

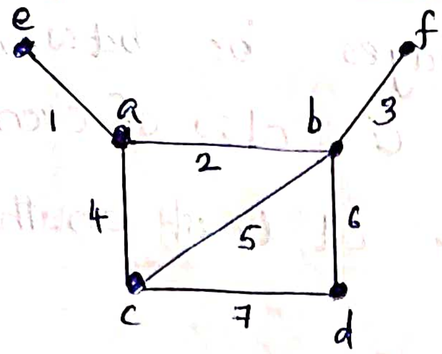
Eg:

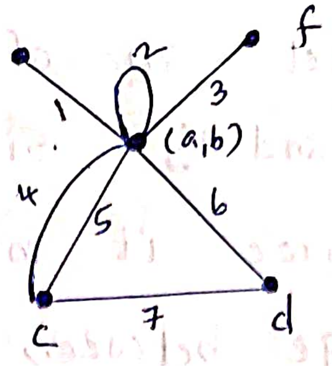


VI FUSION

A pair of vertices a, b in a graph are said to be fused (merged or identified) if the two vertices are replaced by a single new vertex 's' and every edge that was incident on either on a or b or on both is incident on the new vertex.

Note: Fusion doesn't alter number of edges but reduces the number of vertices by one.





Fusion of a & b.

Theorem 2.2

A connected graph G is an Euler graph iff it can be decomposed into circuits.

Proof

Suppose that G can be decomposed into circuits. i.e, G is the union of edge disjoint circuits.

Since the degree of every vertex in a circuit is two, the degree of every vertex in G is even. Hence G is an Euler graph.

Conversely, let G be an Euler graph. Consider a vertex v_1. There are at least two edges incident with v_1.

Let one of these edges be between v_1 and v_2. Since v_2 is also of even degree it must have at least another edge between v_2 & v_3

Proceeding like this gradually we arrive at a vertex that has previously been traversed, thus forming a circuit. C.

Let us remove C from G. All vertices in the remaining graph (not necessarily connected) must also be of even degree.

From the remaining graph remove another circuit in exactly same way as we removed C from G.

Continue this process untill no edges are left. Hence an Euler graph can be decomposed into circuits.

Hence the theorem.

HAMILTUNIAN PATHS AND CIRCUITS

Hamiltonian Circuit (cycle)

Hamiltonian Circuit in a connected graph is defined as a closed walk that traverses every vertex of G exactly once, except the starting vertex at which the walk terminates.

Hamiltonian Path

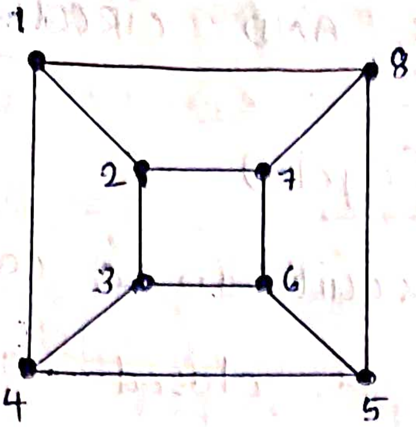
It is a path in the graph which contains every vertex of G exactly once. (i, open walk)

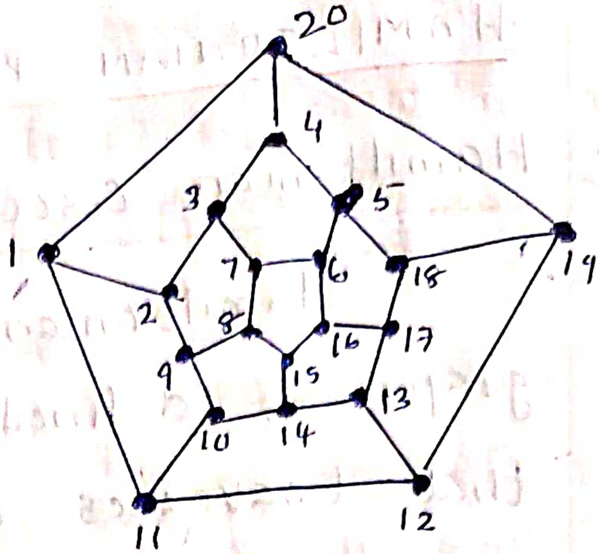
If we remove any one edge from a Hamiltonian circuit we are left with a Hamiltonian path.

Hamiltonian Graph

A graph G is called Hamiltonian if it has a Hamiltonian cycle.

Note: There are no known criterion we can apply to determine the existance of a Hamiltonian circuit in general.

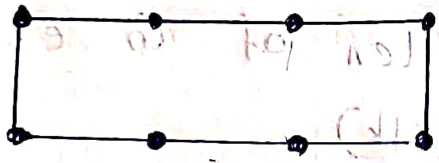




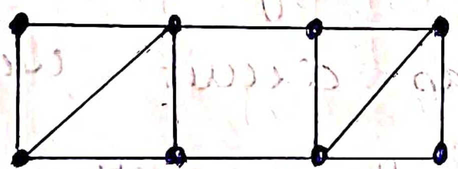
Dodecahedron
Graph with 20 vertices
30 edges.

Qn. Check which of them are Hamiltonian, Euler or both.

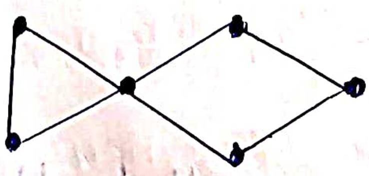
1)



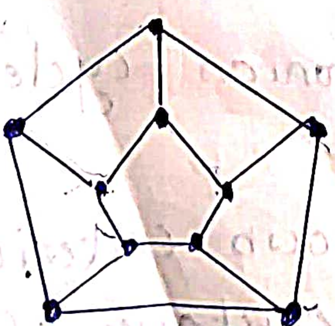
2).



3)



4)



5) Peterson's graph

Qn. What general class of graphs is guaranteed to have a Hamiltonian circuits?

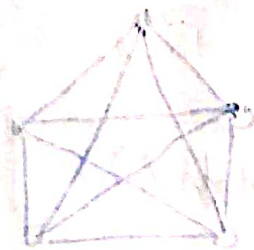
Complete graph (Universal graph) (clique) of 3 or more vertices has Hamiltonian circuits. i.e., $K_3, K_4, K_5, \ldots\ldots$

<u>Note:</u>

1 - Hamiltonian circuits cannot include loops or parallel edges.

The length of the Hamiltonian path in a connected graph with n vertices is $n-1$

Before looking for a Hamiltonian cycle, graph may be made simple by removing loops & parallel edges.



Theorem

In a complete graph with n vertices there are $(n-1)/2$ edge disjoint Hamiltonian circuits if n is an odd number $(n \geq 3)$

Proof

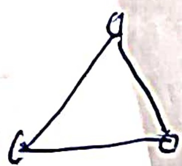
A complete graph with n vertices has exactly $\frac{n(n-1)}{2}$ edges.

A Hamiltonial cycle in a graph with n vertices contains n edges.

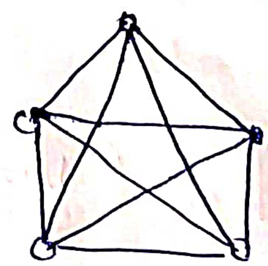
Hence the number of edge disjoint cycles in a Hamiltonian cycle does not exceed $\frac{n-1}{2}$

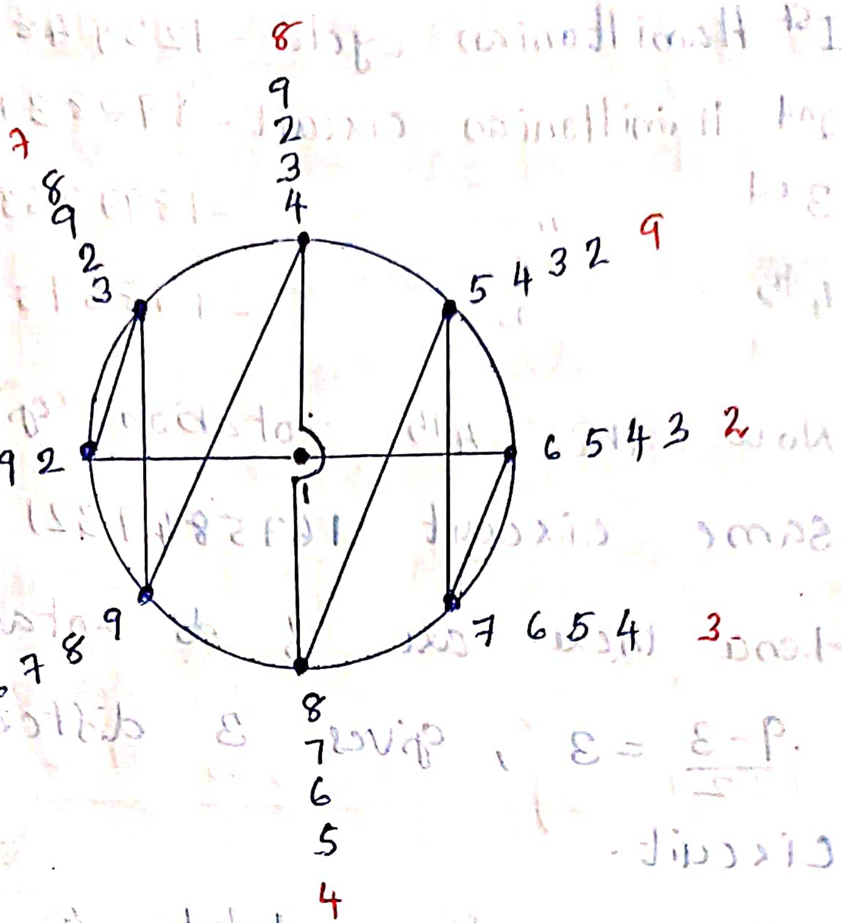
Now we illustrate that if n is an odd number $(n \geq 3)$ there are $\frac{n-1}{2}$ edge disjoint Hamiltonian Circuits

K_3



K_5





Consider the complete graph with 9 vertices & K_9, $n=9$

Consider a Hamiltonian circuit with 9 vertices keeping the vertices fixed (fix one vertex at the centre) on a circle.

Rotate the polygon pattern clockwise by $1 \cdot \frac{2\pi}{8}$, $2 \cdot \frac{2\pi}{8}$, $3 \cdot \frac{2\pi}{8}$

Each rotation produces a Hamiltonian cycle that has no edge in common with any of the previous one.

1st Hamiltonian cycle - 1 2 3 9 4 8 5 7 6 1

rotation: 2nd Hamiltonian circuit - 1 9 2 8 3 7 4 6 5 1

3rd " - 1 8 9 7 2 6 3 5 4 1

4th " - 1 7 8 6 9 5 2 4 3 1

Now the 4th rotation gives the same circuit 1 6 7 5 8 4 9 3 2 1 as first one.

Hence there are 3 rotations & $\frac{9-3}{2} = 3$, gives 3 different Hamiltonian circuit.

Hence we have total 4 Hamiltonian circuits i.e. $\frac{n-1}{2} = \frac{9-1}{2} = 4$

Hence in general $\frac{(n-3)}{2} \times \frac{2\pi}{(n-1)}$ rotations produces $\frac{n-3}{2}$ Hamiltonian circuits. Hence total $\frac{n-3}{2} + 1 = \frac{n-1}{2}$ Edge disjoint Hamiltonian circuits are there.

(For K_5, $1 \cdot \frac{2\pi}{4}$, $2 \cdot \frac{2\pi}{4}$, rotations gives 2 different Hamiltonian circuits

Hence $\frac{5-1}{2} = 2$, edge disjoint circuits are present.) [verify]

Seating Arrangement Problem

Nine members of a club meets each day for lunch at a round table. They decide to sit such that every member has different neighbours at each lunch. How many days can the arrangement lasts.

Solution:

2

- Represent a member by a vertex
- Possibity of sitting next to another member by an edge between them.
- Since every member is allowed to sit near to any other member G is a complete graph with 9 vertices
- 9 is the number of people to be seated around the table
- Every seating arrangement around the table is clearly a Hamiltonian circuit.

- By the above theorem the number of edge disjoint Hamiltonian Cycle is $\frac{n-1}{2} = \frac{8}{2} = 4$.

i.e, 4 such arrangement exists among 9 peoples

Travelling Salesman Problem (TSP)

Problem:

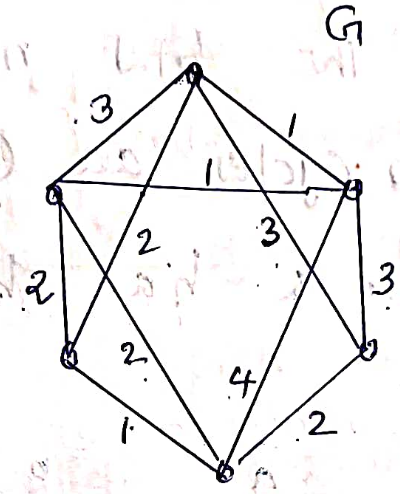
A salesman is required to visit a number of cities during a trip. Given the distance between the cities. In what order should he travel so as to visit every city precisely once and return home, with the minimum mileage travelled.

Before giving the solution for TSP we will see the following definition and result.

Weighted graph

A weighted graph is a graph in which each edge e has been assigned a real numbers say $w(e)$ called the weight or length of e.

Eg:



Here $w(G) = 24$ (sum of weights of edges of G)

Result

The total numbers of different Hamiltonian circuits in a complete graph of n vertices is $\frac{(n-1)!}{2}$

Proof

This follows from the fact that starting from any vertex we have $(n-1)$ edges to move forward.

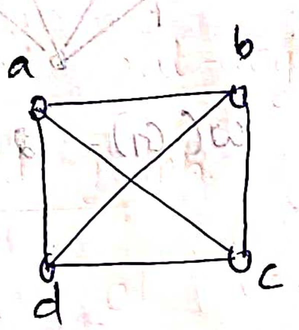
$(n-2)$ ~~ver~~ edges from the second vertex to move forward $(n-3)$ edges from the third vertex and so on.

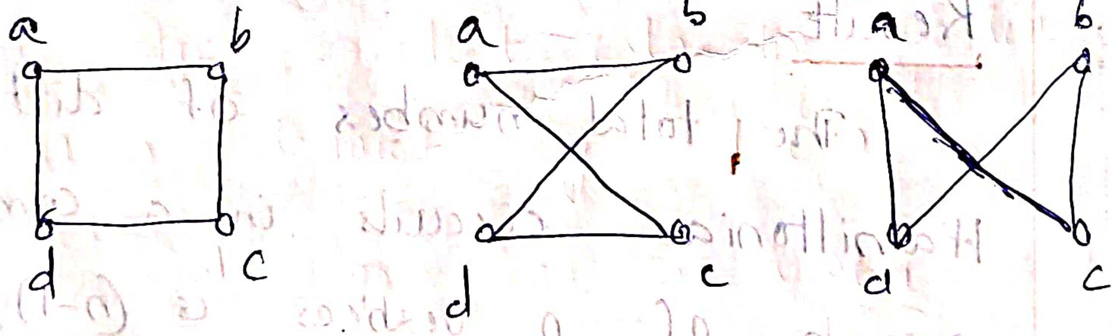
$\therefore$ Total number of choices are

$(n-1)(n-2)(n-3)\cdots\times 1 = (n-1)!$

Now the total number of different Hamiltonian cycles are $\frac{(n-1)!}{2}$ since each cycle has been counted twice.

Eg: K_4





There are $\frac{(4-1)!}{2} = \frac{3!}{2} = 3$ different Hamiltonian cycle as above

Solution to TSP

Represent the cities by vertices and the roads between them by edges. In this graph with every edge e_i there is associated a real number $w(e_i)$, the distance in miles.

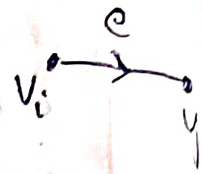
Here in this graph we have $\frac{(n-1)!}{2}$ different Hamiltonian circuit and we will pick up the one that has the smallest sum of distances.

Directed graphs (Digraphs / Oriented graphs)

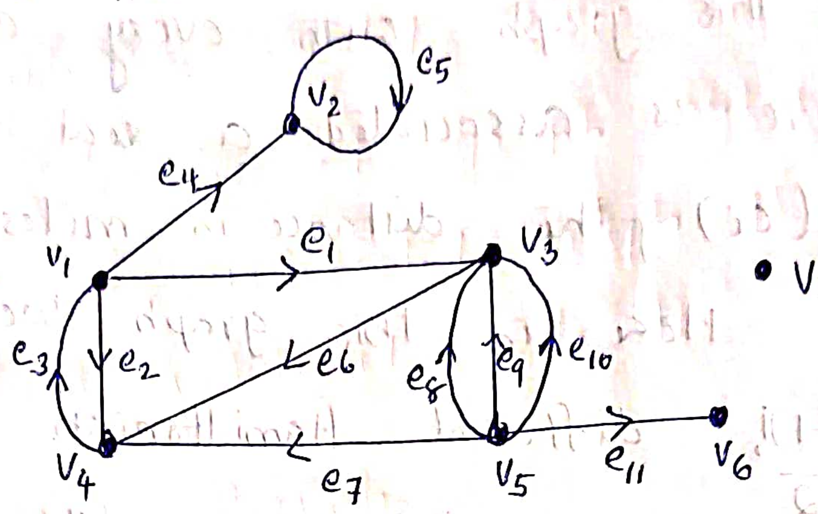
A directed graph G consists of a set of vertices $V = \{v_1, v_2, v_3, \ldots\}$ and a set of edges, $E = \{e_1, e_2, e_3, \ldots\}$ where every edge e_i is represented by some ordered pair of vertices (v_i, v_j)

As in the case of undirected graphs a vertex is represented by a point and an edge by a line

segment between v_i and v_j with an arrow directed from v_i to v_j.
v_i is the initial vertex and v_j is the terminal vertex



Eg:



In-degree and out-degree of a vertex in a directed graph.

In-degree of vertex is the number of edges incident into the vertex v and is denoted by $d^-(v)$

Out-degree of a vertex v is the number of edges incident out of a vertex v, and is denoted by $d^+(v)$

A directed loop contributes one to both in degree & out-degree of a vertex

- In the above example,

$d^-(v_1) = 1$ $\quad$ $d^+(v_1) = 3$

$d^-(v_2) = 2$ $\quad$ $d^+(v_2) = 1$

$d^-(v_3) = 4$ $\quad$ $d^+(v_3) = 1$

$d^-(v_4) = 3$ $\quad$ $d^+(v_4) = 1$

$d^-(v_5) = 0$ $\quad$ $d^+(v_5) = 5$

$d^-(v_6) = 1$ $\quad$ $d^+(v_6) = 0$

$d^-(v_7) = 0$ $\quad$ $d^+(v_7) = 0$

Total → 11 $\quad$ Total → 11

Result:

$$\sum_{i=1}^{n} d^-(v_i) = \sum_{i=1}^{n} d^+(v_i) = \text{number of edges}$$

i.e, Sum of in-degree = Sum of out-degree.

- For an isolated vertex in a digraph indegree & out-degree are zero.

- For a pendant vertex in a digraph
$d^+(v) + d^-(v) = 1$

- Two edges in a digraph is said to be parallel directed edges if they have same initial vertex and terminal vertex & same direction.

Undirected graph corresponding to G

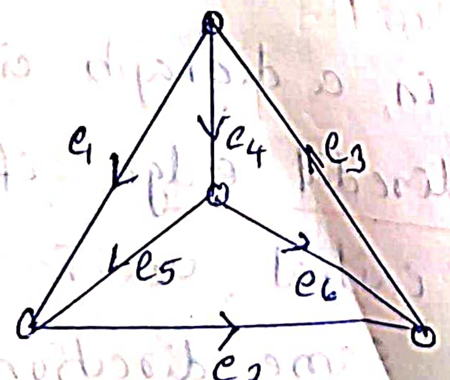
It is the undirected graph obtained from a directed graph. G.

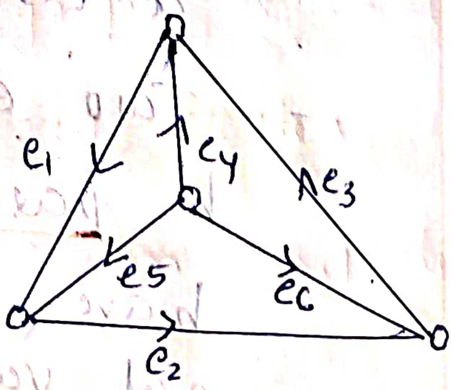
Digraph associated with H

A directed graph obtained from an undirected graph H is called digraph associated with H.

Isomorphic Digraphs

Two digraphs are said to be isomorphic if they have identical behaviour in terms of graph properties. i.e, there should be a one - one correspondance between vertex set, edge set, direction between two graphs.





Both the graphs are nonisomorphic since e_4 has different directions.

Some types of Digraphs

1) Simple digraphs

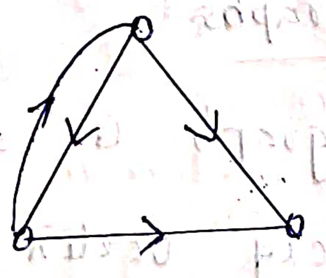
A digraph that has no self loops and parallel edges.

2. Asymmetric or Antisymmetric digraphs

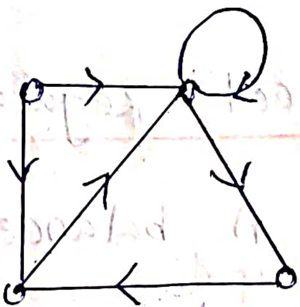
Digraphs that have atmost one directed edge between a pair of vertices but allowed to have self loops.

3. Symmetric diagraphs

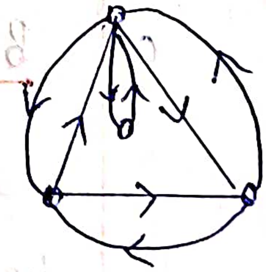
Digraphs in which for every edge (a,b) there is also an edge (b,a)



Simple digraph



Asymmetric digraphs

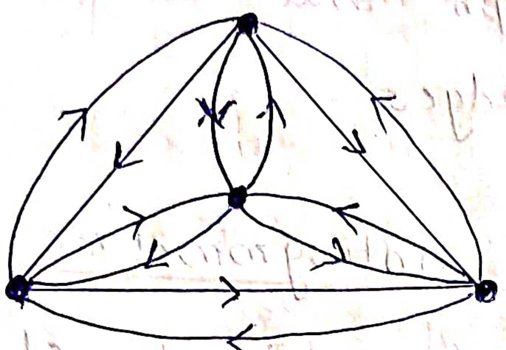


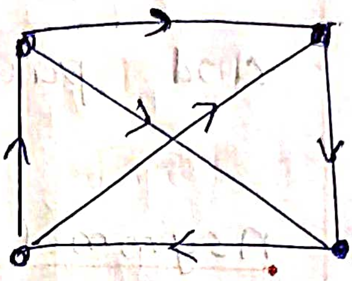
symmetric digraphs

4) Complete Digraphs – two type

Complete Symmetric digraph

Complete antisymmetric digraphs (Tournaments)





No. of edges $= n(n-1)$

$2 \times \frac{n(n-1)}{2}$

No. of edges $= \frac{n(n-1)}{2}$

5) Pseudosymmetric digraphs or Isographs or balanced digraphs.

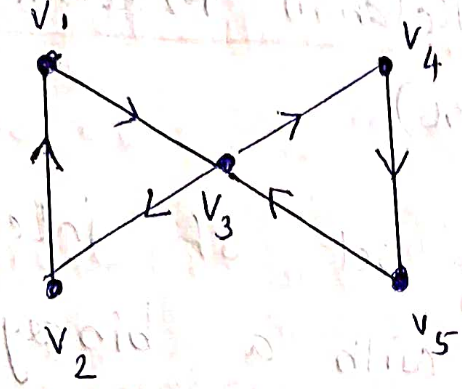
A digraph is said to be balanced if every vertex v_i the indegree equals the out-degree $d^+(v_i) = d^-(v_i)$

6) Balanced Regular digraphs

A balanced digraph is said to be regular if every vertex has the same in-degree and same out-degree

Examples

Balanced digraphs



$d^-(v_1) = 1 = d^+(v_1)$

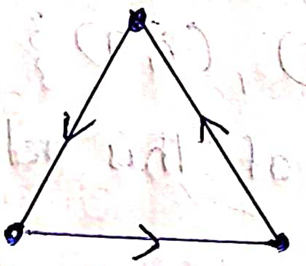
$d^-(v_2) = 1 = d^+(v_2)$

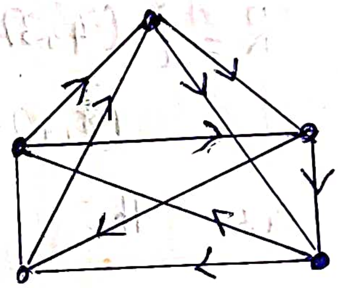
$d^-(v_3) = d^+(v_3) = 2$

$d^-(v_4) = d^+(v_4) = 1$

$d^-(v_5) = d^+(v_5) = 1$

Balanced Regular digraphs





$d^+(v) = d^-(v) = 1$ for all vertices.

Balanced 1-regular digraph.

$d^+(v) = d^-(v) = 2$

Balanced 2-regular digraphs.

Digraphs and Binary Relation

Some binary relations are "is parallel to", "is perpendicular to", "is greater than", "is equal to", "is a factor of" and so on.

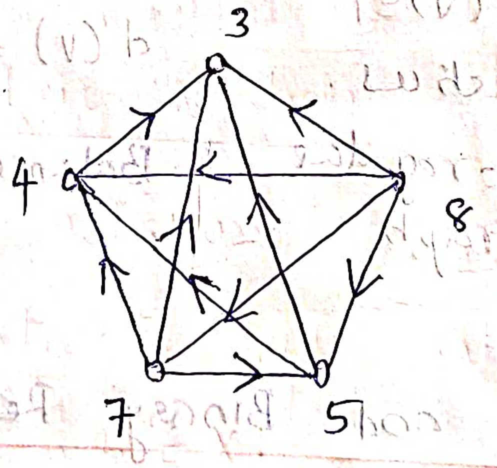
Representation of a binary relation as a digraph & (Relation Matrix or Adjacency matrix)

Consider the set of integers $\{3, 4, 7, 5, 8\}$ with a binary relation "is greater than"

$$R = \{(4,3), (7,3), (5,3), (8,3), (5,4), (7,4), (8,4), (7,5), (8,5), (8,7)\}$$

are the elements of this relation.

Digraph of the above binary relation,



[Eg. for transitive digraph]

Note: Every binary relation on a finite set can be represented by a digraph without parallel edges.

Relation Matrix

It is an $n \times n$ square matrix where n is the number of elements. The $(i,j)^{th}$ entry in the matrix is '1' if $x_i R x_j$ and is '0' otherwise.

$$
\begin{array}{c}
 \\ 3 \\ 4 \\ 7 \\ 5 \\ 8
\end{array}
\begin{array}{c}
\begin{array}{ccccc} 3 & 4 & 7 & 5 & 8 \end{array} \\
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 \\
1 & 1 & 0 & 1 & 0 \\
1 & 1 & 0 & 0 & 0 \\
1 & 1 & 1 & 1 & 0
\end{bmatrix}
\end{array}
$$

Reflexive Relation

The relation R on a set X, that satisfy $x_i R x_i$, $\forall x_i \in X$

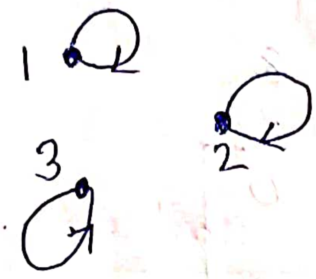
The digraph of a reflexive relation will have a self loop at every vertex

eg: parallel to, equal to etc.

Reflexive digraph

A digraph representing reflexive binary relation on its vertex set.

Eg: $X = \{1, 2, 3\}$, with relation " is equal



Irreflexive digraph

A digraph in which there is no self loop in its vertex set.

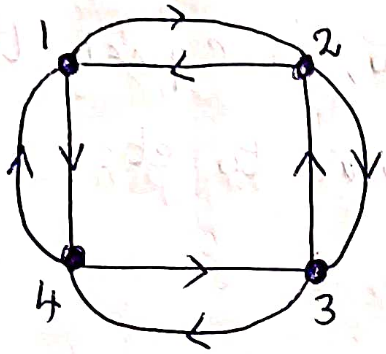
Symmetric Relation

A relation R on a set X that satisfies if aRb then bRa for all $a, b \in X$.

Symmetric Digraph

A digraph representing symmetric binary relation is called symmetric digraph. Every directed edge from vertex v_i to v_j, there is a directed edge from vertex v_j to v_i.

Eg: For an irreflexive symmetric relation on 4 elements.



It can also be represented as,

where each undirected edge represents two parallel but oppositely directed edge.

Transitive Relation

A relation R is said to be transitive if for any three elements a, b, c belongs to the set if aRb and bRc, then aRc.

Eg: Is greater than

Transitive digraph

A digraph representing transitive relation on its sets of vertices is called a transitive digraph. (example in previous page)

Equivalance Relation

A binary relation is called equivalence Relation if it is reflexive, symmetric and transitive.

Eg: Is parallel to, is equal to, is isomorphic to etc

Equivalence graph

A graph representing an equivalence relation is an equivalence graph.

Eg: $X = \{10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20\} \equiv \bmod(3)$

Relation is congruent modulo 3 i.e,

$a \equiv b \pmod 3 \Rightarrow 3 / a-b$

Now, partitions are

$[0], [1], [2]$

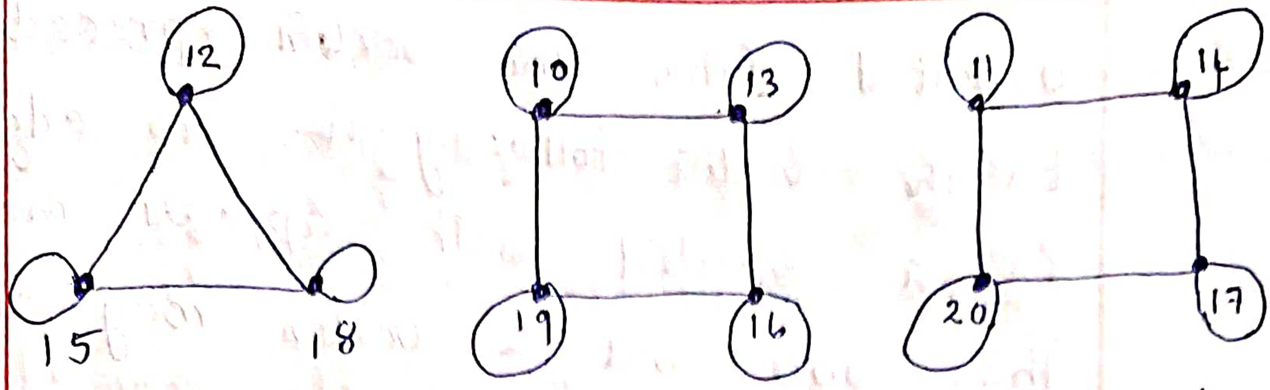
$[0] = \{12, 15, 18\}$, $[1] = \{10, 13, 16, 19\}$

$[2] = \{11, 14, 17, 20\}$

$[0]$ - elements with remainder ~~[?]~~ 0

$[1]$ - elements with remainder 1

$[2]$ - elements with remainder 2



Each undirected edge represents two parallel but oppositely directed edges.

Directed Paths, Walks & Circuits

Walks, paths & circuits in a directed graph, in addition to being what they are in the undirected graph, have the added consideration of orientation or direction.

Walk in a digraph (Directed walk)

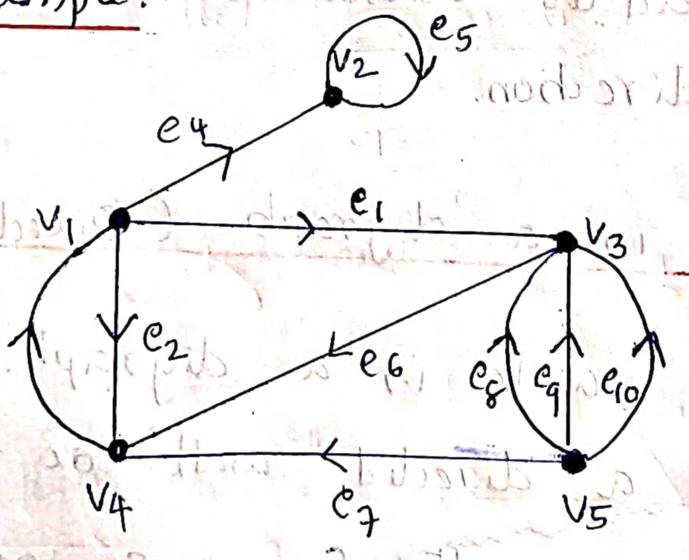
A walk in a digraph can mean either a directed walk or semi walk

Directed walk from a vertex v_i to v_j is an alternating sequence of vertices and edges, beginning with v_i and ending with v_j such that each edge is oriented from the vertex preceding it to the vertex following it. No edge in a directed walk appears more than once, but a vertex may appear more than once.

<u>Semi walk</u> in a directed graph is a walk in the corresponding undirected graph, but is not a directed walk.

Illy we have <u>directed paths, & semipaths, directed circuits and semi circuit.</u> in digraphs.

<u>Example:</u>



Directed walk : $v_1 e_1 v_3 e_6 v_4 e_3 v_1 e_4 v_2$

Semi walk : $v_1 e_1 v_3 e_9 v_5 e_7 v_4 e_2 v_1 e_3 v_4$

(No orientation)

Directed path : $v_5 e_9 v_3 e_6 v_4 e_3 v_1$

semi path : $v_5 e_8 v_3 e_1 v_1$ (no orientation)

Directed circuit : $v_1 e_1 v_3 e_6 v_4 e_3 v_1$

semi circuits : $v_1 e_3 v_4 e_2 v_1$ (no orientation)

Connected digraphs (2 types)

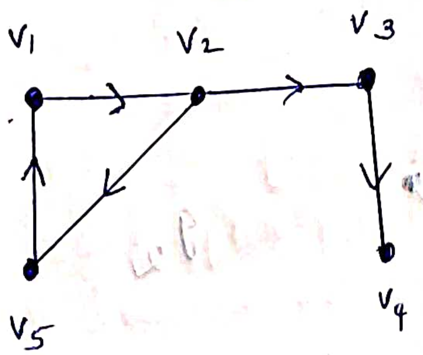
Strongly connected digraphs

If there is at least one directed path from every vertex to every other vertex then the digraph G is said to be strongly connected.

Weakly connected digraphs

A digraph is said to be weakly connected if its corresponding undirected graph is connected.

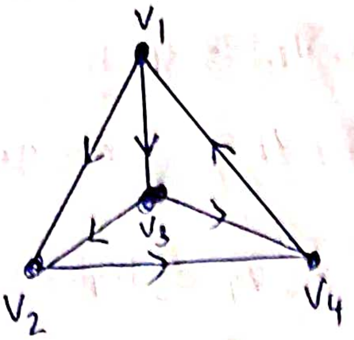
Eg. 1)



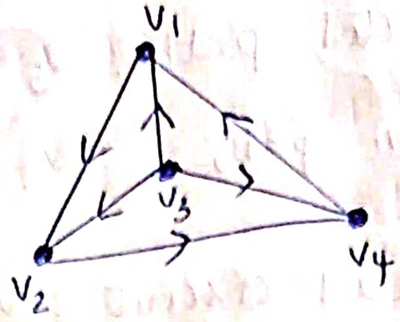
Not strongly connected

But weakly connected

3)



strongly connected graph



weekly connected graph

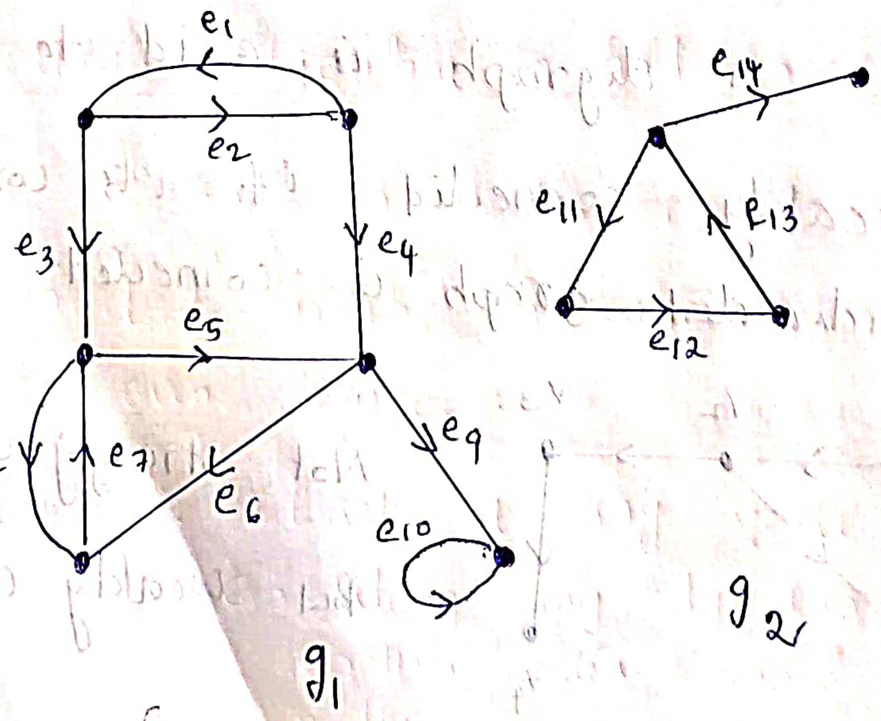
Component of a digraph

Each maximal connected (weakly or strongly) subgraph of a digraph G is called a component of G.

This maximal strongly connected subgraphs within each component will be called fragments.

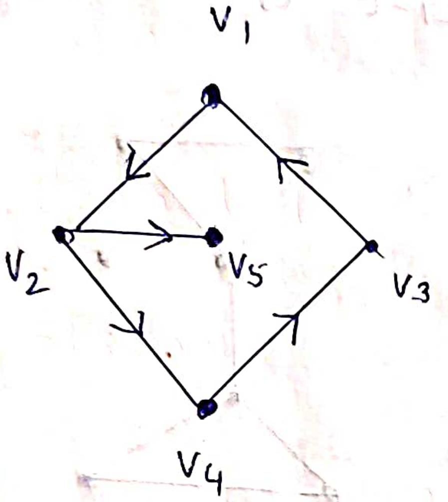
G

Eg:

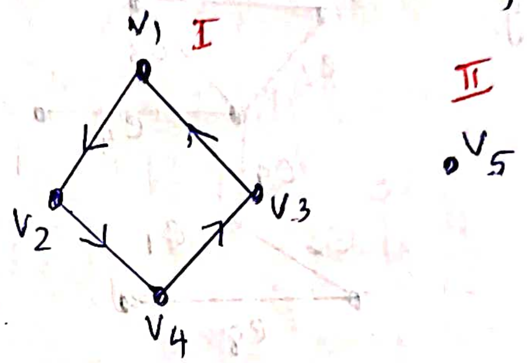


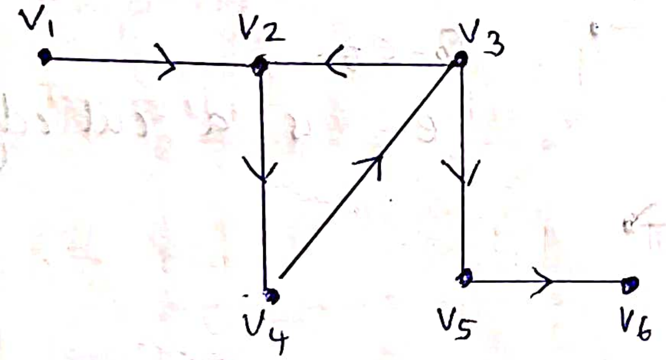
g_1 & g_2 are two components of graph G

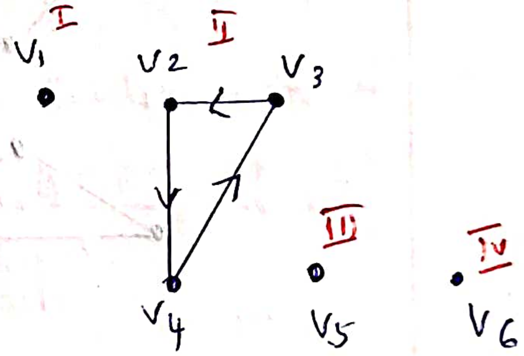
In g_1, $\{e_1, e_2\}$, $\{e_5, e_6, e_7, e_8\}$, $\{e_{10}\}$ are three fragments.



2 strongly connected components.





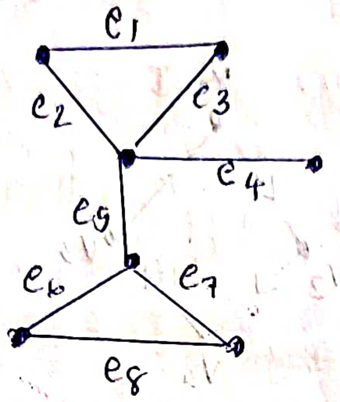


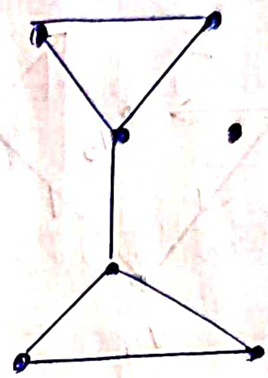
Four strongly connected components

Cut edge or Bridge

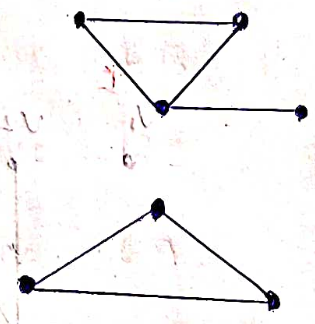
An edge of a graph is called a bridge (or a cut edge) if the subgraph $G-e$ is disconnected.

Eg:





$G-e_4$, e_4 is bridge



$G-e_5$

e_5 is a cut edge

Fleury's Algorithm to find an Euler Circuit

To find an Euler circuit in any connected graph in which each vertex has even degree.

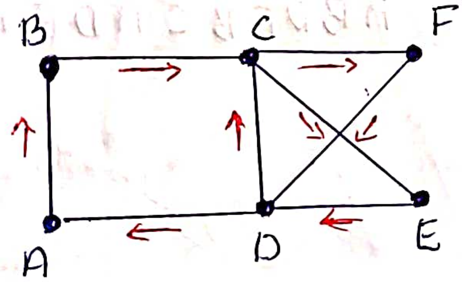
Step I : Start at any vertex

Go along any edge from this vertex to another vertex. Remove this edge from the graph.

Step 2 : You are now on a vertex on the revised graph. choose any edge from this vertex but not a cut edge unless you have no other option. Remove this edge from the graph

Step 3 : Repeat step 2 untill you have used all the edges and return back to the vertex at which you started.

Qn) Find an Euler circuit using Fleury's algorithm.

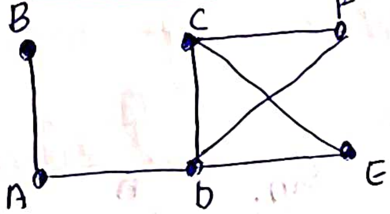


Euler circuit is

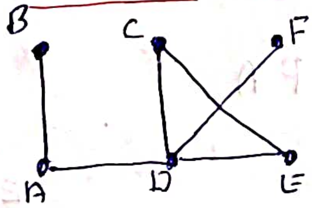
BCFDCEDAB

start from B

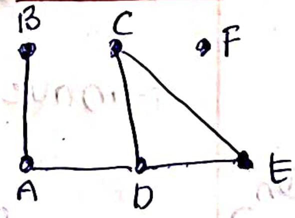
Remove BC



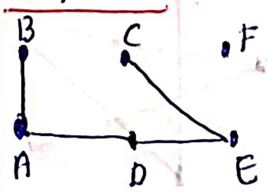
Remove CF



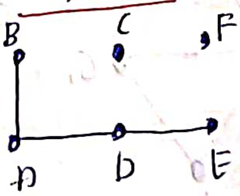
Remove FD



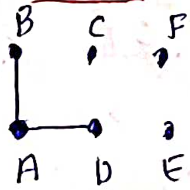
Remove DC



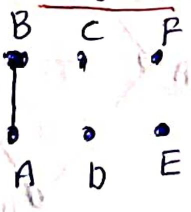
Remove CE



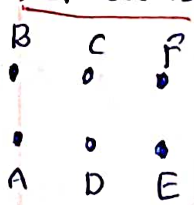
Remove ED



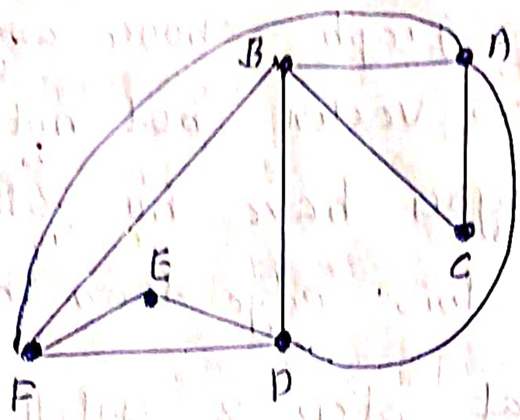
Remove DA



Remove AB



(2)



Starting from vertex A,

Remove AB

Remove BD

Remove DF

Remove FB

Remove BC

Remove CA

Remove AD

Remove DE

Remove EF

Remove FA

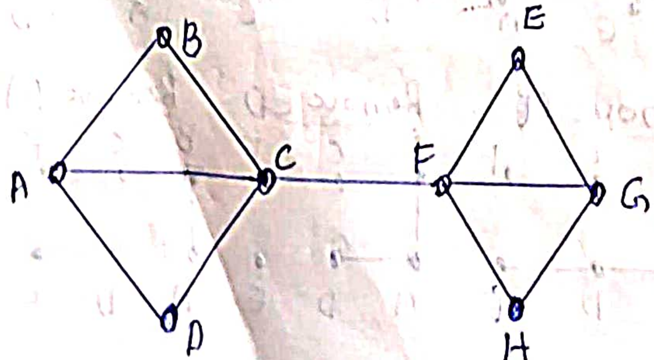
Euler circuit is

ABDFBCADEFA.

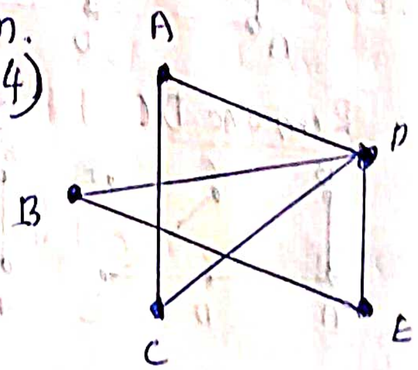
tw)

-

Qn. 3)

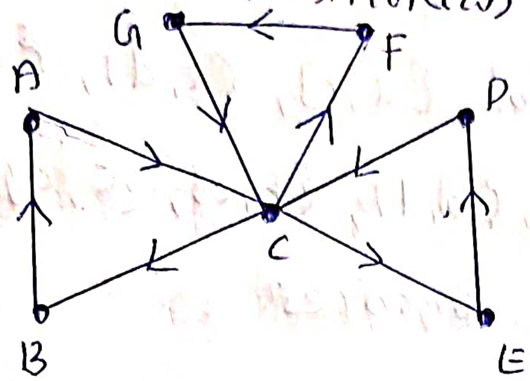


Qn. 4)



MCQ Questions

1) Give an example of a strongly connected simple digraph without a directed Hamiltonian path.



Does not contain a Hamiltonian path but every vertices are connected by a directed path hence strongly connected.

2. Let S be a set of 5 elements. Construct a graph G whose vertices are subsets of S with size 2 and two such subsets are adjacent in G if they are disjoint.

1) Draw the graph G

2) How many edges must be added to G in order, for G to have a Hamiltonian cycle.

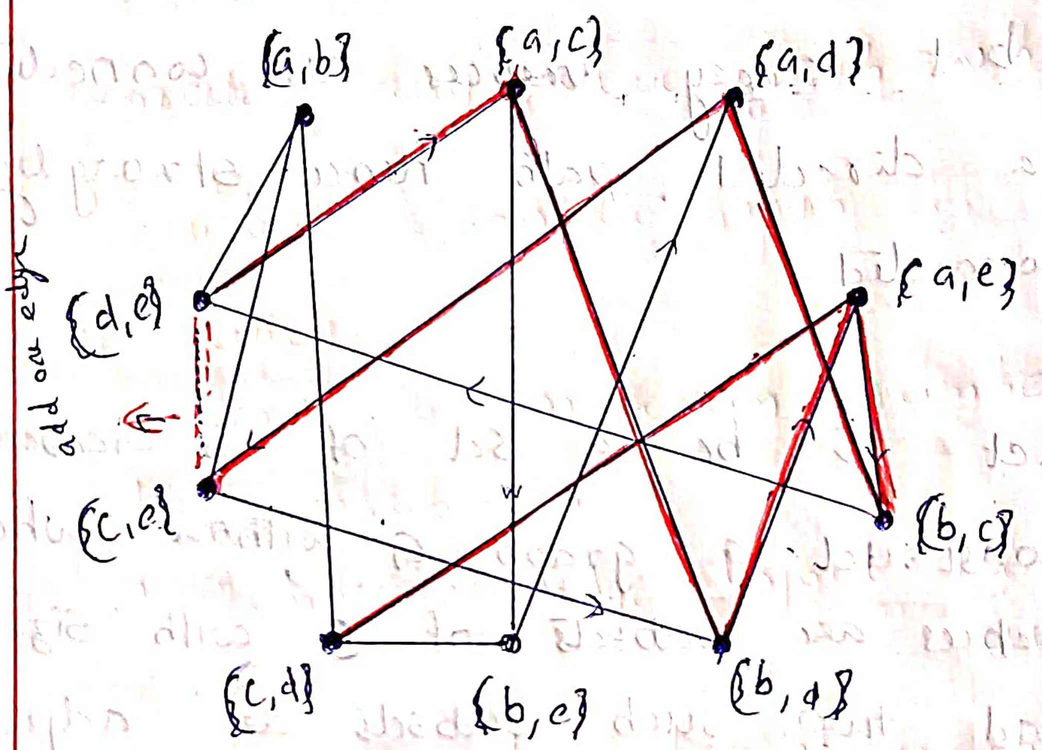
Let $S = \{a, b, c, d, e\}$

vertices of G are subsets of S with size 2.

i, $V = \{\{a,b\}, \{a,c\}, \{a,d\}, \{a,e\},$
$\{b,c\}, \{b,d\}, \{b,e\}, \{c,d\}, \{c,e\}, \{d,e\}$

Total 10 vertices.

G.



Join those vertices by an edge Those who have no common elements.

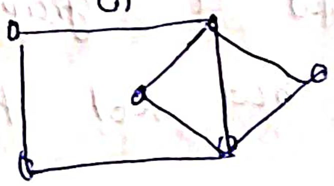
One edge must be added to G inorder to have a Hamiltonian cycle.

3) Let G be a graph with exactly two connected components both being Eulerian. What is the minimum number of edges that need to be added to G to obtain an Eulerian Graph.

4) Let G be a graph with exactly two connected components both being Hamiltonian graphs. Find the minimum numbers of edges that one needs to add to G to obtain a hamiltonian graph.

5) Differentiate between symmetric and assymmetric ~~gra~~ digraphs with examples and draw a Complete symmetric digraph of 4 vertices.

6) G



Define Euler graph. Is G Euler? If yes write an Euler Line for G. Write necessary & sufficient condition for G to be Euler & prove it.

3) Define Hamiltonian circuit & paths with example. Find out the number of edge disjoint Hamiltonian circuit possible in a complete graph with 5 vertices.

4. State TSP and how TSP is related with Hamiltonian circuit.

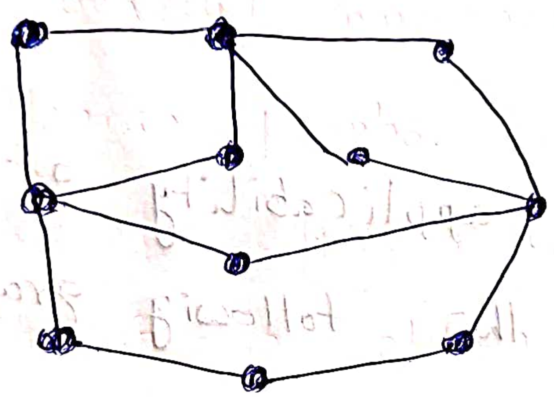
5. Differentiate btw. complete symmetric & complete assymmetric graph with examples.

6. Consider a complete graph G with 11 vertices.
1) Find the maximum number of edges possible in G
2) Number of edge disjoint Hamiltonian circuit

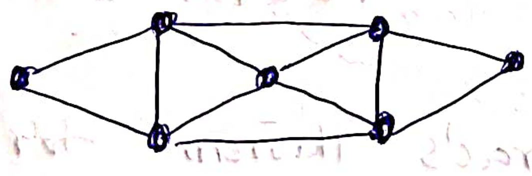
7 Which general class of graph is guaranteed to have a Hamiltonian circuits. Also draw a graph that has Hamiltonian path but not Hamiltonian circuits.

8) Draw a connected graph that becomes disconnected when only one edge is removed from it.

9) Check whether the the given graph is Euler. If yes Give an Euler line, Justify your answer.



10. State TSP. Give a travelling salesman Tour on the graph below (means Hamiltonian circuit)



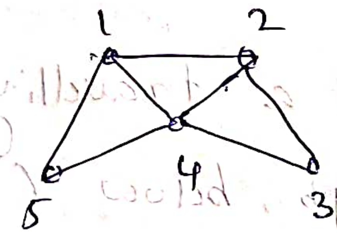
12 Prove or disprove if every vertex of a simple graph G has degree two, then G is a cycle.

Dirac's Theorem for Hamiltonianicity

(sufficient condition only not necessary)

If G is a simple graph with n vertices $n \geq 3$, and $d(v_i) \geq n/2$ for every vertex v of G then G is Hamiltonian.

Qn. Check the applicability of Dirac's theorem in the following graph.



$d(v) \geq n/2$ for all vertices. $n = 5$, $n/2$

$\therefore G$ is hamiltonian

12345 1 is a H. circuit.

Qn. State Dirac's theorem for Hamiltonian circuit and why it is not a necessary condition for a simple graph to have a Hamiltonian cycle.

(Give counter example)